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# FMP: Toward Fair Graph Message Passing against Topology Bias

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Topology bias refers to the fact that in many graphs, nodes with the same sensitive attribute (attribute besides node features, e.g., age, region information, nationality, etc) are more likely to get connected. This paper [1] theoretically proved that the aggregation in message passing inevitably accumulates representation bias when large topology bias exists. Integrated in a unified optimization framework, the fair message passing (FMP) scheme is proposed to simultaneously guarantee graph smoothness and enhance fairness. Specifically, the optimization problem is as follows:

$$\min_{\mathbf{F}} \frac{\lambda_s}{2} \text{tr}(\mathbf{F}^T \mathbf{L} \mathbf{F}) + \frac{1}{2} \|\mathbf{F} - \mathbf{X}\| + \lambda_f \|\Delta_s \text{softmax}(\mathbf{F})\|_1.$$

The sensitive attribute is defined as  $\mathbf{s} \in \{-1, 1\}^n$  and the sensitive attribute incident vector is defined as  $\Delta_s = \frac{\mathbb{I}_{>0}(\mathbf{s})}{\|\mathbb{I}_{>0}(\mathbf{s})\|_1} - \frac{\mathbb{I}_{>0}(-\mathbf{s})}{\|\mathbb{I}_{>0}(-\mathbf{s})\|_1}$ . The  $\text{softmax}(\mathbf{F})$  represents the predicted probability for node classification task, where  $[\text{softmax}(\mathbf{F})]_{ij} = P(y_i = j | \mathbf{X})$ . Therefore, we can conclude that

$$\begin{aligned} [\Delta_s \text{softmax}(\mathbf{F})]_j &= \frac{\sum_{\mathbf{s}_i=1} P(y_i = j | \mathbf{X})}{\|\mathbb{I}_{>0}(\mathbf{s})\|_1} - \frac{\sum_{\mathbf{s}_i=-1} P(y_i = j | \mathbf{X})}{\|\mathbb{I}_{>0}(-\mathbf{s})\|_1} \\ &= P(y_i = j | \mathbf{s}_i = 1, \mathbf{X}) - P(y_i = j | \mathbf{s}_i = -1, \mathbf{X}). \end{aligned}$$

In other words, the last term characterizes the predicted probability difference between two groups with sensitive attribute. The optimization problem can be solved via Fenchel conjugate and proximal alternating predictor-corrector algorithm, which is quite similar to the one used in ElasticNet. Intuitively, the proposed method ensure that the predicted node label distribution does not vary much across different node sets with different sensitive attributes.

## References

- [1] Zhimeng Jiang, Xiaotian Han, Chao Fan, Zirui Liu, Na Zou, Ali Mostafavi, and Xia Hu. Fmp: Toward fair graph message passing against topology bias. *arXiv preprint arXiv:2202.04187*, 2022. (document)